Single-view Metrology and Cameras

Computational Photography
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Slides adopted from Derek Hoiem
Review: Pinhole Camera

\[
P = \begin{bmatrix} u \\ v \end{bmatrix}
\]

Optical Center \((u_0, v_0)\)

Camera Center \((t_x, t_y, t_z)\)

\[
P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]
Review: Projection Matrix

$$\mathbf{x} = \mathbf{K}[\mathbf{R} \hspace{0.1cm} \mathbf{t}] \mathbf{X}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f & s & u_0 \\ 0 & cf & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$
Take-home question review

• Suppose the camera axis is in the direction of \((x=0, y=0, z=1)\) in its own coordinate system. What is the camera axis in world coordinates given the extrinsic parameters \(R, t\)?

• Suppose a camera at height \(y=h\) \((x=0, z=0)\) observes a point at \((u,v)\) known to be on the ground \((y=0)\). Assume \(R\) is identity. What is the 3D position of the point in terms of \(f, u_0, v_0\)?
Take-home question review

Suppose we have two 3D cubes on the ground facing the viewer, one near, one far.

1. What would they look like in perspective?
2. What would they look like in weak perspective?
Review: Vanishing Points

Vertical vanishing point (at infinity)

Vanishing line

Vanishing point

Vanishing point

Slide from Efros, Photo from Criminisi
This class

• How can we calibrate the camera?
• How can we measure the size of objects in the world from an image?
• What about other camera properties: focal length, field of view, depth of field, aperture, f-number?
• How to do “focus stacking” to get a sharp picture of a nearby object
• How the “vertigo effect” works
How to calibrate the camera?

\[ x = K [R \kern2pt t] X \]

\[
\begin{bmatrix}
  wu \\
  ww \\
  w
\end{bmatrix}
= \begin{bmatrix}
  * & * & * & * \\
  * & * & * & * \\
  * & * & * & * \\
\end{bmatrix}
\begin{bmatrix}
  X \\
  Y \\
  Z \\
  1
\end{bmatrix}
\]
Calibrating the Camera

Method 1: Use an object (calibration grid) with known geometry

– Correspond image points to 3d points
– Get least squares solution (or non-linear solution)

\[
\begin{bmatrix}
wu \\
wv \\
w
\end{bmatrix} =
\begin{bmatrix}
m_{11} & m_{12} & m_{13} & m_{14} \\
m_{21} & m_{22} & m_{23} & m_{24} \\
m_{31} & m_{32} & m_{33} & m_{34}
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]
Calibrating the Camera

Method 2: Use vanishing points

– Find vanishing points corresponding to orthogonal directions

Vanishing line

Vertical vanishing point (at infinity)

Vanishing point
Take-home question (for later)

Suppose you have estimated finite three vanishing points corresponding to orthogonal directions:

1) How to solve for intrinsic matrix? (assume K has three parameters)
   - The transpose of the rotation matrix is its inverse
   - Use the fact that the 3D directions are orthogonal

2) How to recover the rotation matrix that is aligned with the 3D axes defined by these points?
   - In homogeneous coordinates, 3d point at infinity is \((X, Y, Z, 0)\)
How can we measure the size of 3D objects from an image?
Perspective cues
Perspective cues
Perspective cues
Ames Room
Comparing heights

Vanishing Point
Measuring height

Camera height
Two views of a scene

Parallel to ground

Camera center

Image horizon

Image

Slight foreshortening due to camera angle
Which is higher – the camera or the parachute?
Measuring height without a giant ruler

Compute $Z$ from image measurements
  - Need a reference object
The cross ratio

A Projective Invariant

• Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points

\[ \frac{\|P_3 - P_1\|}{\|P_3 - P_2\|} \frac{\|P_4 - P_2\|}{\|P_4 - P_1\|} \]

Can permute the point ordering

• 4! = 24 different orders (but only 6 distinct values)
This is the fundamental invariant of projective geometry

\[ P_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix} \]
Measuring height

\[
\frac{\|B - T\|}{\|R - T\|} = \frac{H}{R}
\]

scene cross ratio

\[
\frac{\|b - t\|}{\|v_z - r\|} = \frac{H}{R}
\]

image cross ratio

scene points represented as 
\[ P = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \]

image points as 
\[ p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \]

Slide by Steve Seitz
Measuring height

vanishing line (horizon)

\[ v \approx (b \times b_0) \times (v_x \times v_y) \]

\[ t \approx (v \times t_0) \times (r \times b) \]

\[ \frac{\|t - b\|}{\|v_z - r\|} = \frac{\|r - b\|}{\|v_z - t\|} \]

image cross ratio
Measuring height

What if the point on the ground plane \( b_0 \) is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find \( b_0 \) as shown above

vanishing line (horizon)
Take-home question

Assume that the man is 6 ft tall
– What is the height of the front of the building?
– What is the height of the camera?
Beyond the pinhole: What about focus, aperture, DOF, FOV, etc?

Camera Center \((t_x, t_y, t_z)\) 

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = P
\]

Optical Center \((u_0, v_0)\)

\[
p = \begin{bmatrix} u \\ v \end{bmatrix}
\]
Adding a lens

A lens focuses light onto the film
- There is a specific distance at which objects are “in focus”
  - other points project to a “circle of confusion” in the image
- Changing the shape of the lens changes this distance
A lens focuses parallel rays onto a single focal point

- focal point at a distance $f$ beyond the plane of the lens
- Aperture of diameter $D$ restricts the range of rays
The human eye is a camera

- **Iris** - colored annulus with radial muscles

- **Pupil** - the hole (aperture) whose size is controlled by the iris

Figure: Fig. 1.1, Principles of Digital Image Synthesis, Volume 1. Andrew Glassner
Focus with lenses

![Diagram of lens focusing](http://en.wikipedia.org/wiki/File:Lens3.svg)

**Equation for objects in focus**

\[
\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}
\]

The aperture and depth of field

Changing the aperture size or focusing distance affects depth of field
f-number (f/#) = focal_length / aperture_diameter (e.g., f/16 means that the focal length is 16 times the diameter)
When you change the f-number, you are changing the aperture
Depth of Field = range around focused distance that leads to smaller than threshold circle of confusion

Slide source: Seitz

Varying the aperture

Large aperture = small DOF

Small aperture = large DOF

Photo credit: Philip Greenspun
Shrinking the aperture

Why not make the aperture as small as possible?
  – Less light gets through
  – Diffraction effects

Figure: Optics. Eugene Hecht
Shrinking the aperture

Figure: Optics. Eugene Hecht
The Photographer’s Great Compromise

<table>
<thead>
<tr>
<th>What we want</th>
<th>How we get it</th>
<th>Cost</th>
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<tbody>
<tr>
<td>More spatial resolution</td>
<td>Increase focal length</td>
<td>Light, FOV</td>
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<tr>
<td></td>
<td>Decrease focal length</td>
<td>DOF</td>
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<td>Broader field of view</td>
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<td></td>
<td>Lengthen exposure</td>
<td>Temporal Res</td>
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<tr>
<td>More light</td>
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Things to remember

• Can calibrate using grid or VP

• Can measure relative sizes using VP

• Effects of focal length, aperture + tricks
Next class

• Go over take-home questions from today
• Single-view 3D Reconstruction