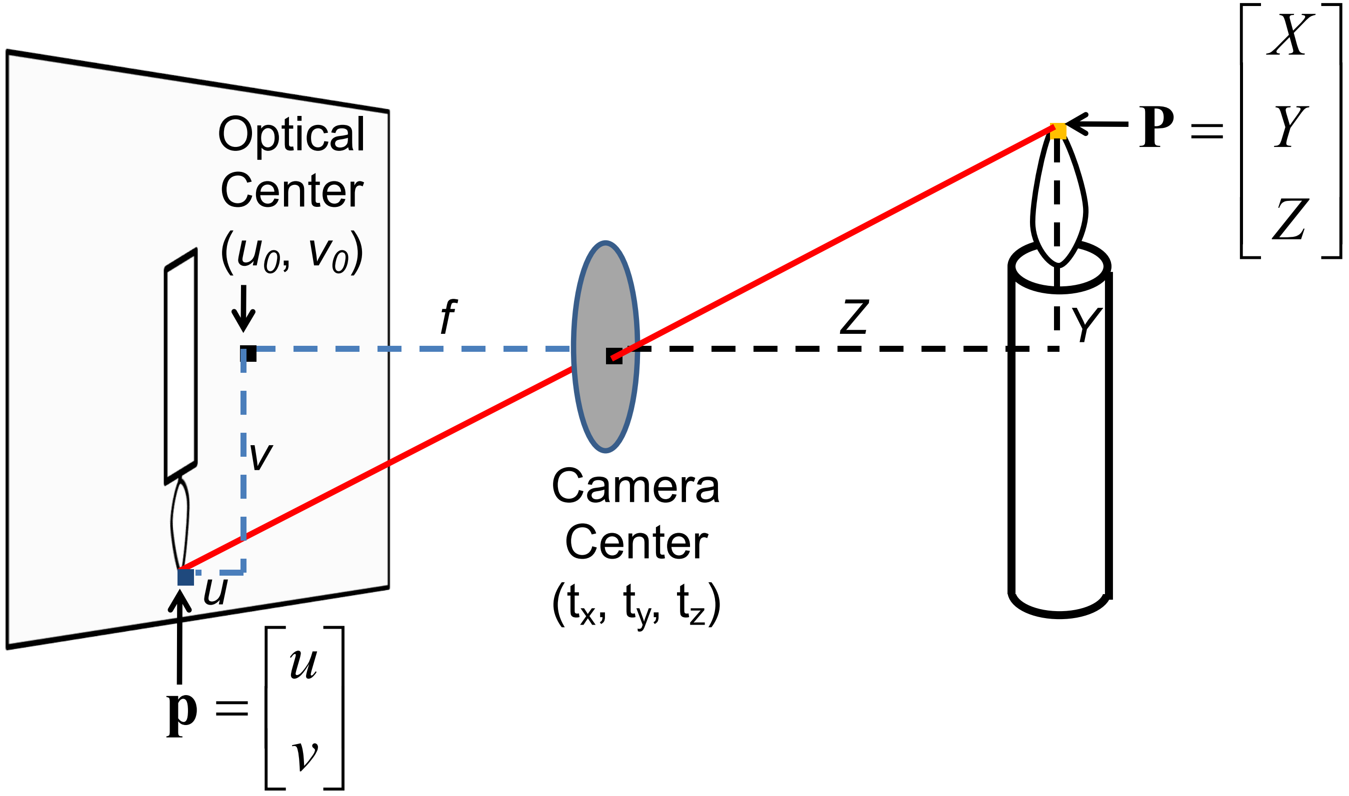
Single-view Metrology and Cameras

Computational Photography

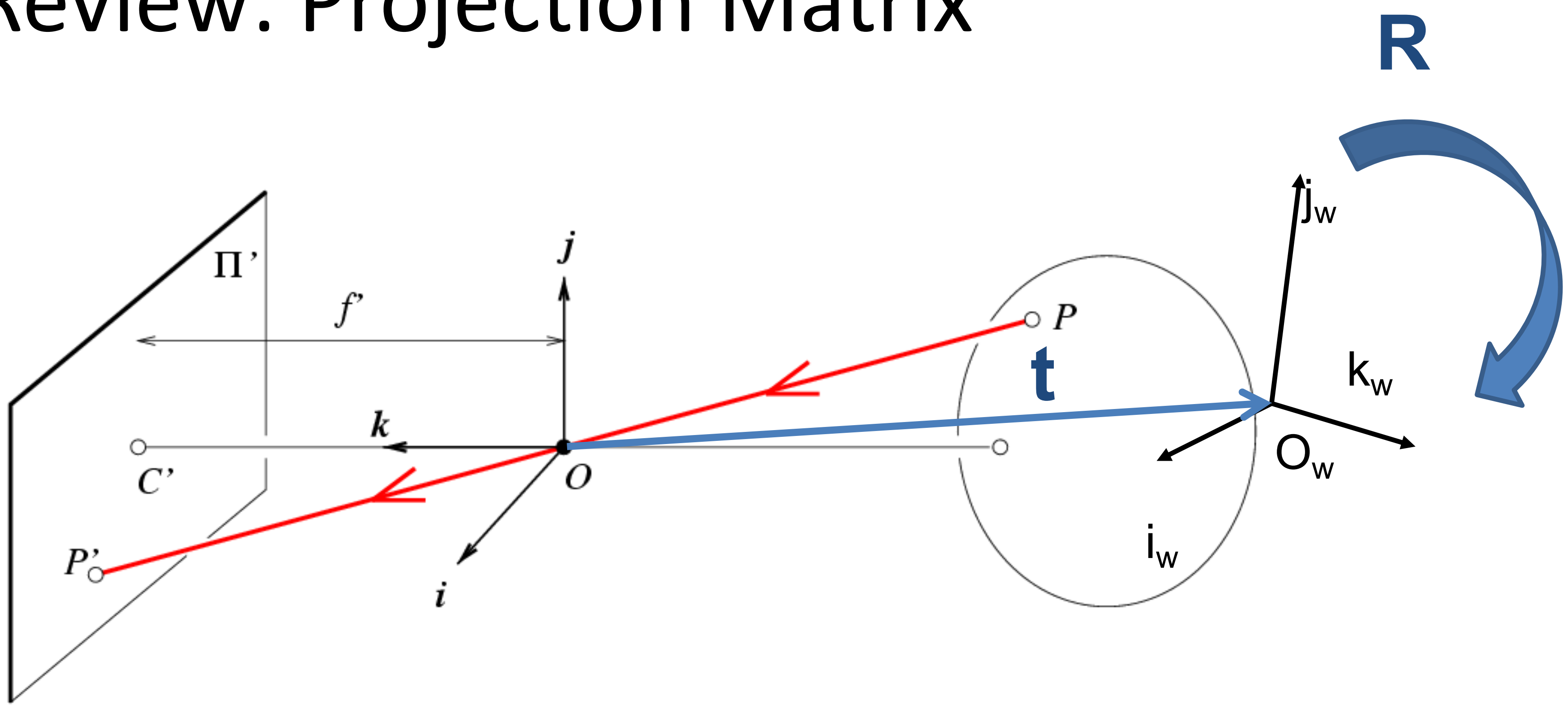
Yuxiong Wang, University of Illinois

Slides adopted from Derek Hoiem

Review: Pinhole Camera



Review: Projection Matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & s & u_0 \\ 0 & \alpha f & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Take-home question review

- Suppose the camera axis is in the direction of (x=0, y=0, z=1) in its own coordinate system. What is the camera axis in world coordinates given the extrinsic parameters ***R***, ***t***

- Suppose a camera at height y=*h (x=0,z=0)* observes a point at *(u,v)* known to be on the ground (*y*=0). Assume *R* is identity. What is the 3D position of the point in terms of f, u_0, v_0?

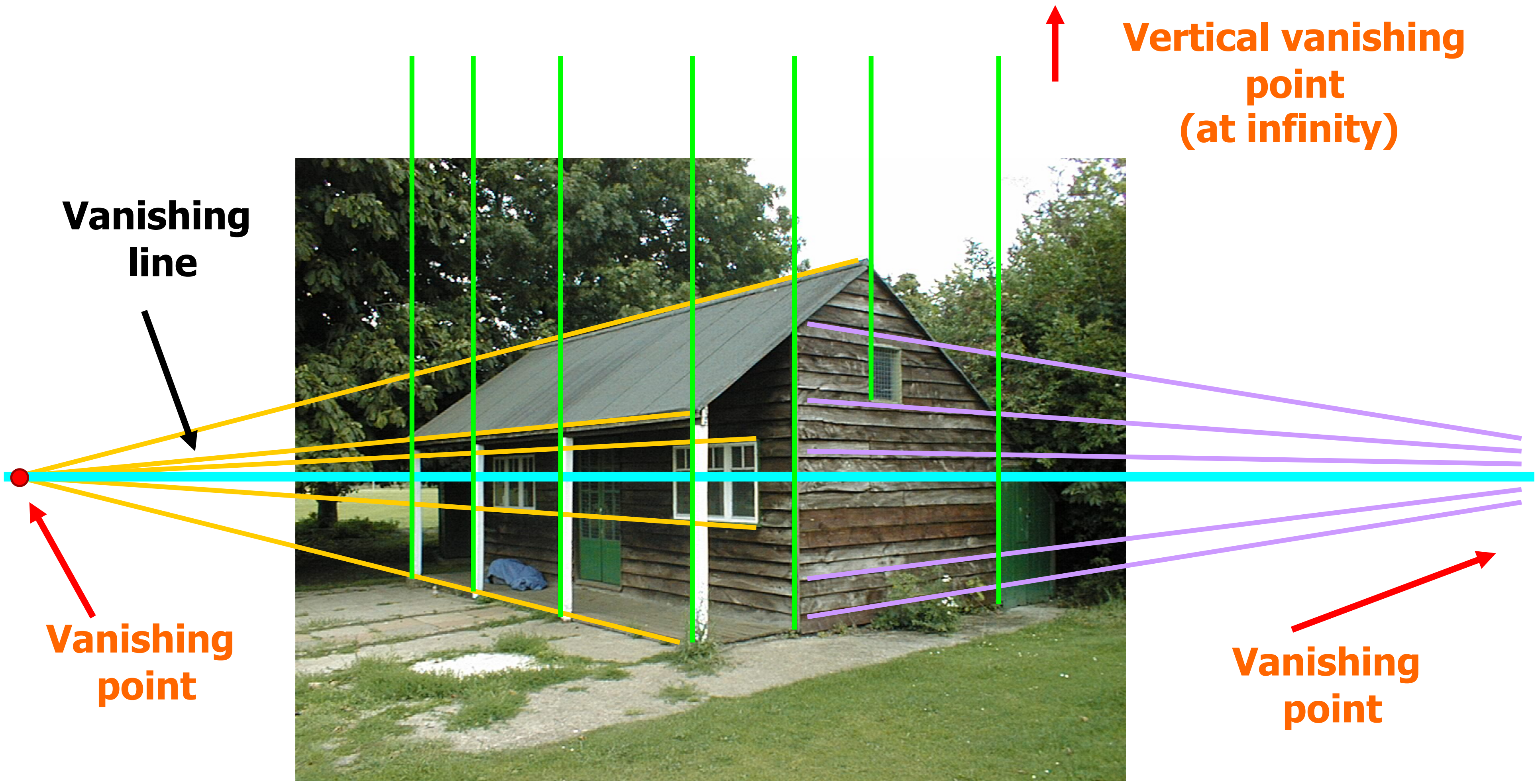
Take-home question review

Suppose we have two 3D cubes on the ground facing the viewer, one near, one far.

1. What would they look like in perspective?
2. What would they look like in weak perspective?

Photo: Kathy from Flickr

Review: Vanishing Points



Slide from Efros, Photo from Criminisi

This class

- How can we calibrate the camera?
- How can we measure the size of objects in the world from an image?
- What about other camera properties: focal length, field of view, depth of field, aperture, f-number?
- How to do “focus stacking” to get a sharp picture of a nearby object
- How the “vertigo effect” works

How to calibrate the camera?

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

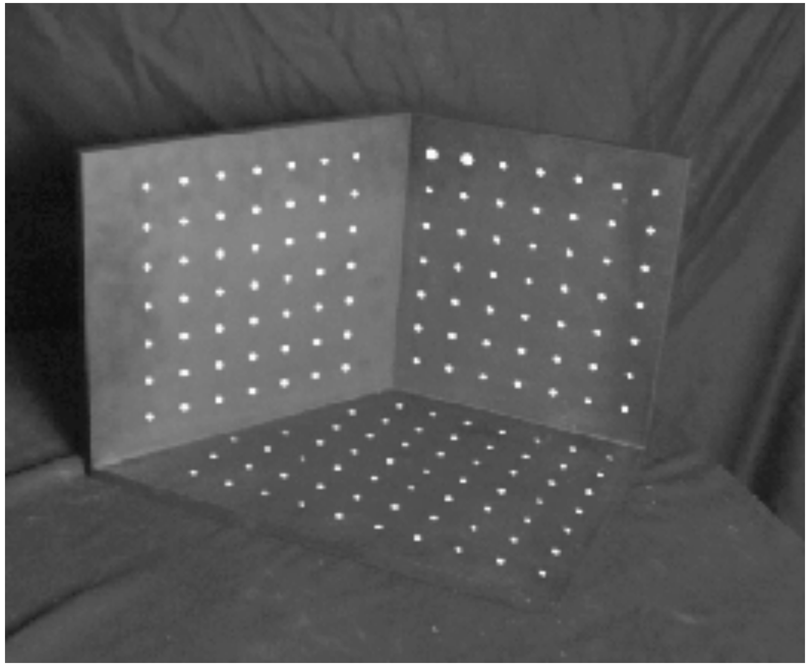
$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Calibrating the Camera

Method 1: Use an object (calibration grid) with known geometry

– Correspond image points to 3d points

– Get least squares solution (or non-linear solution)

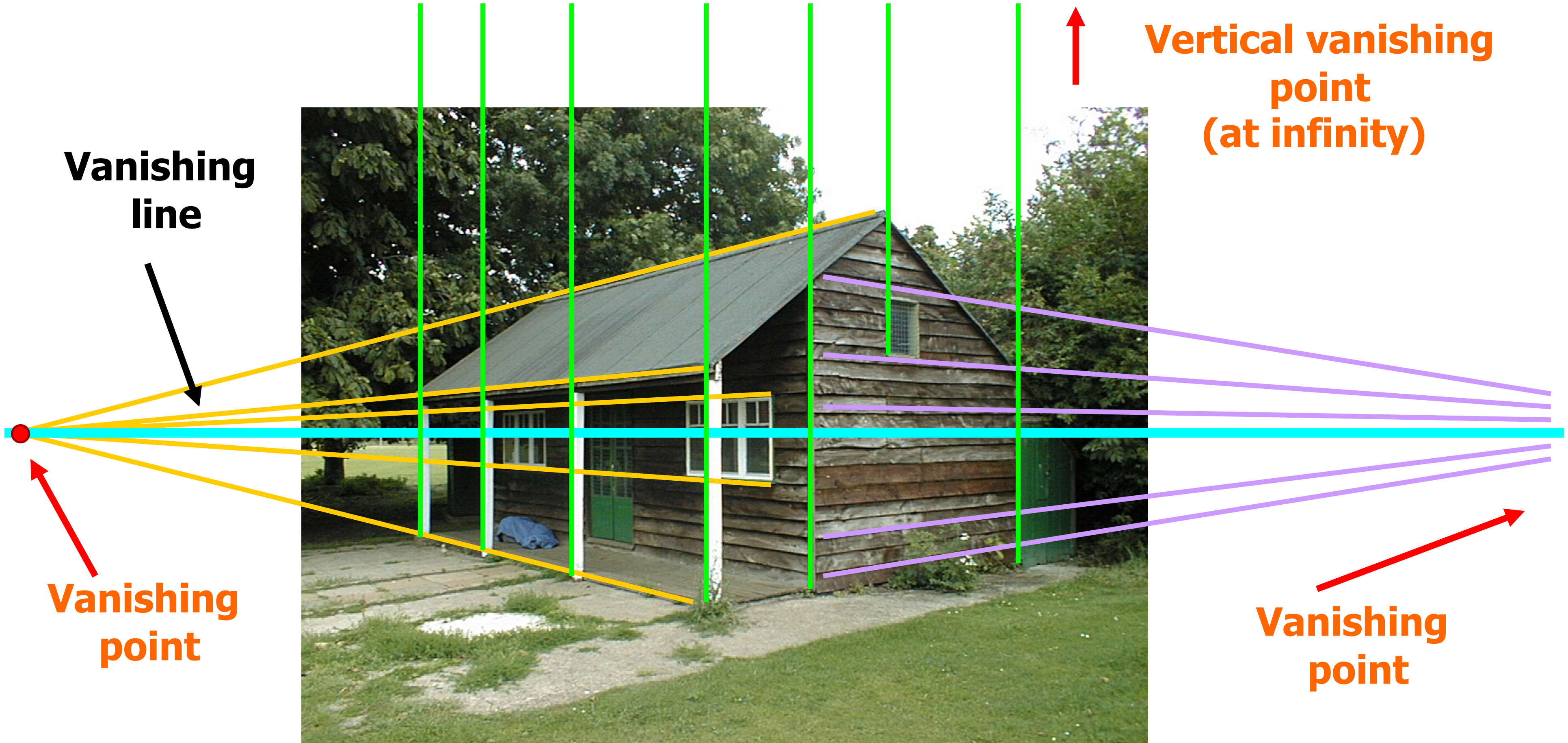


$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Calibrating the Camera

Method 2: Use vanishing points

– Find vanishing points corresponding to orthogonal directions



Take-home question (for later)

Suppose you have estimated finite three vanishing points corresponding to orthogonal directions:

1) How to solve for intrinsic matrix? (assume K has three parameters)
   - The transpose of the rotation matrix is its inverse
   - Use the fact that the 3D directions are orthogonal
2) How to recover the rotation matrix that is aligned with the 3D axes defined by these points?
   - In homogeneous coordinates, 3d point at infinity is (X, Y, Z, 0)

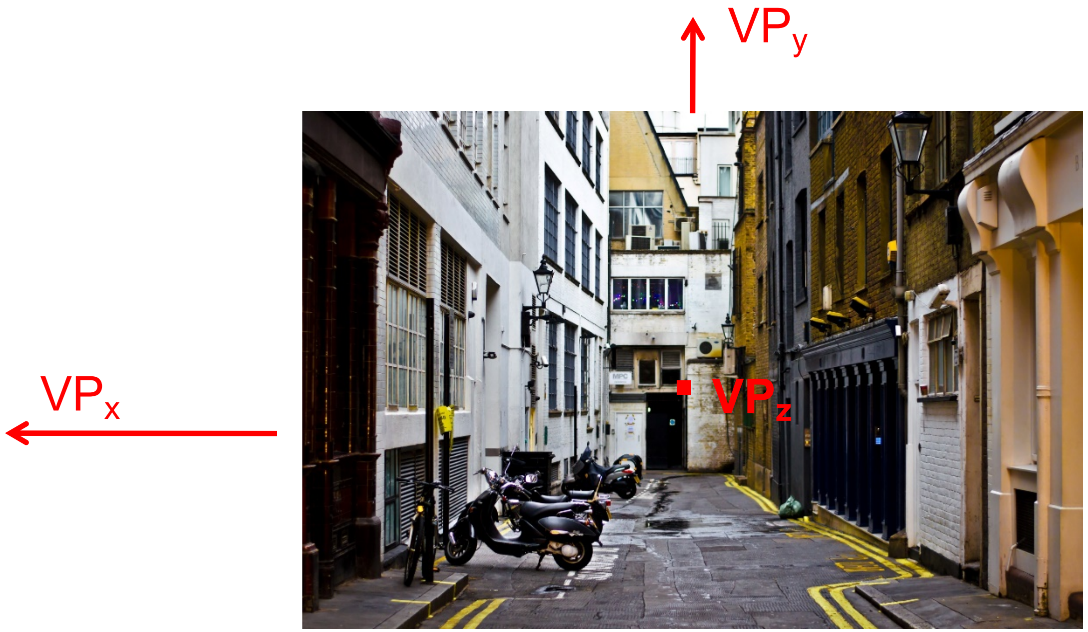
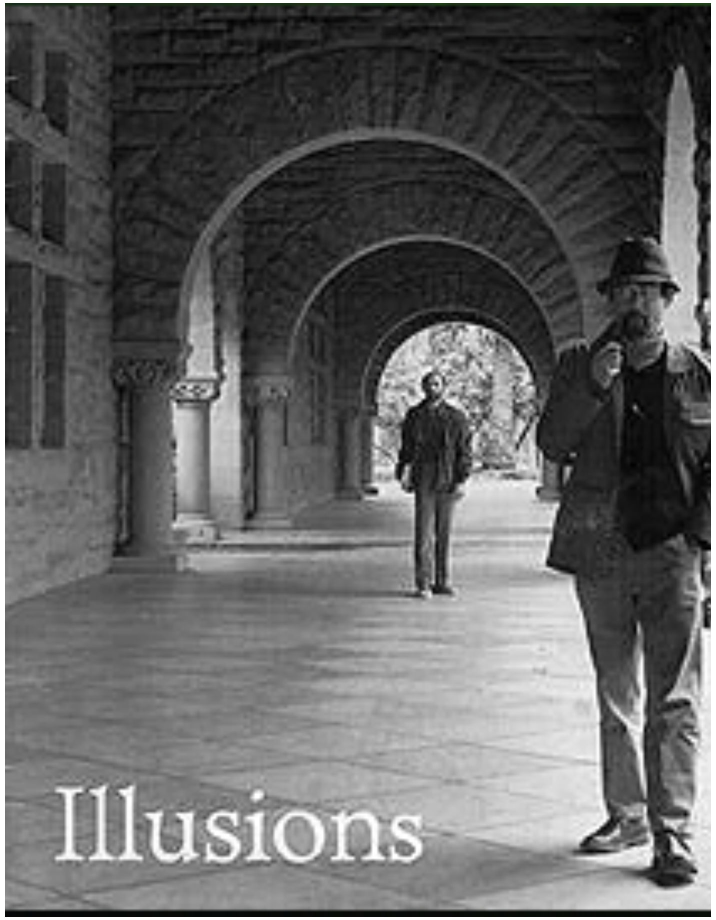
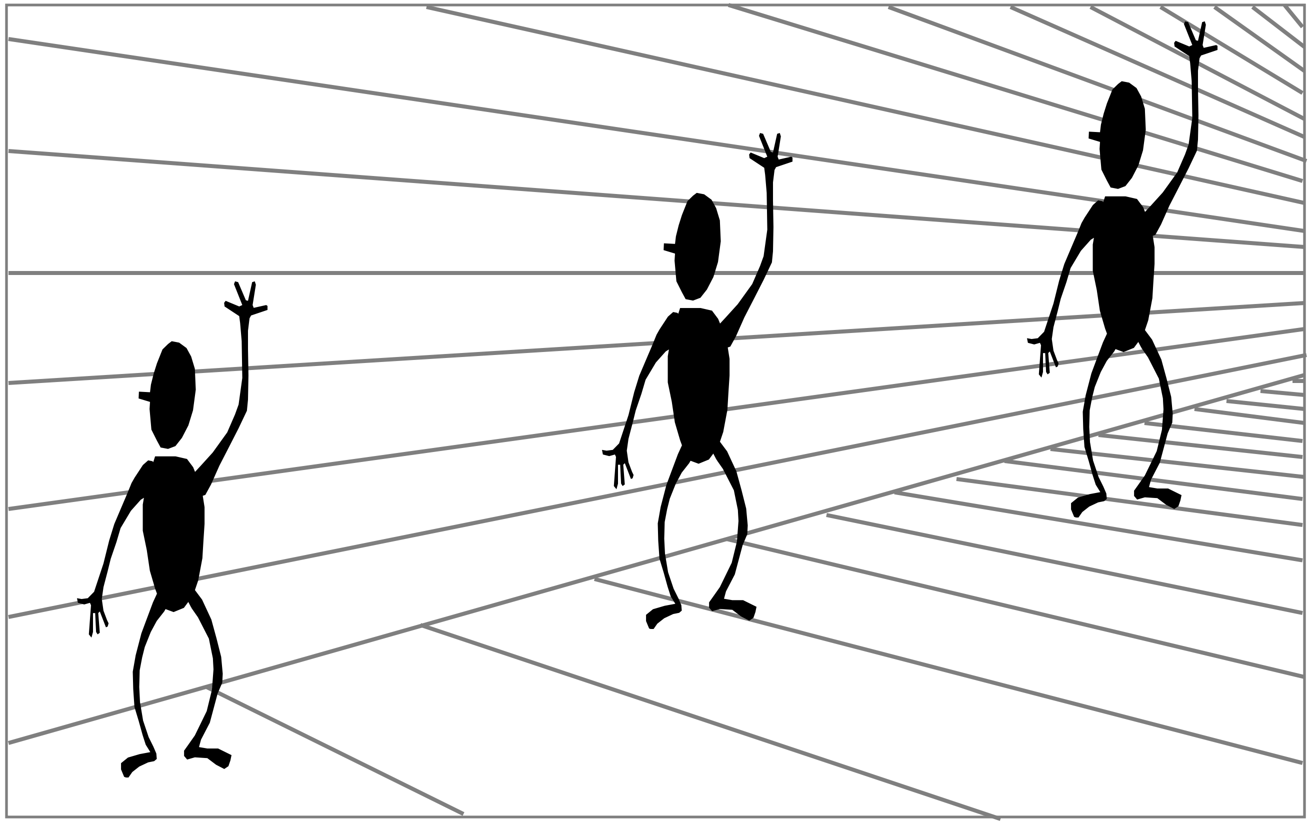


Photo from Garry Knight

How can we measure the size of 3D objects from an image?

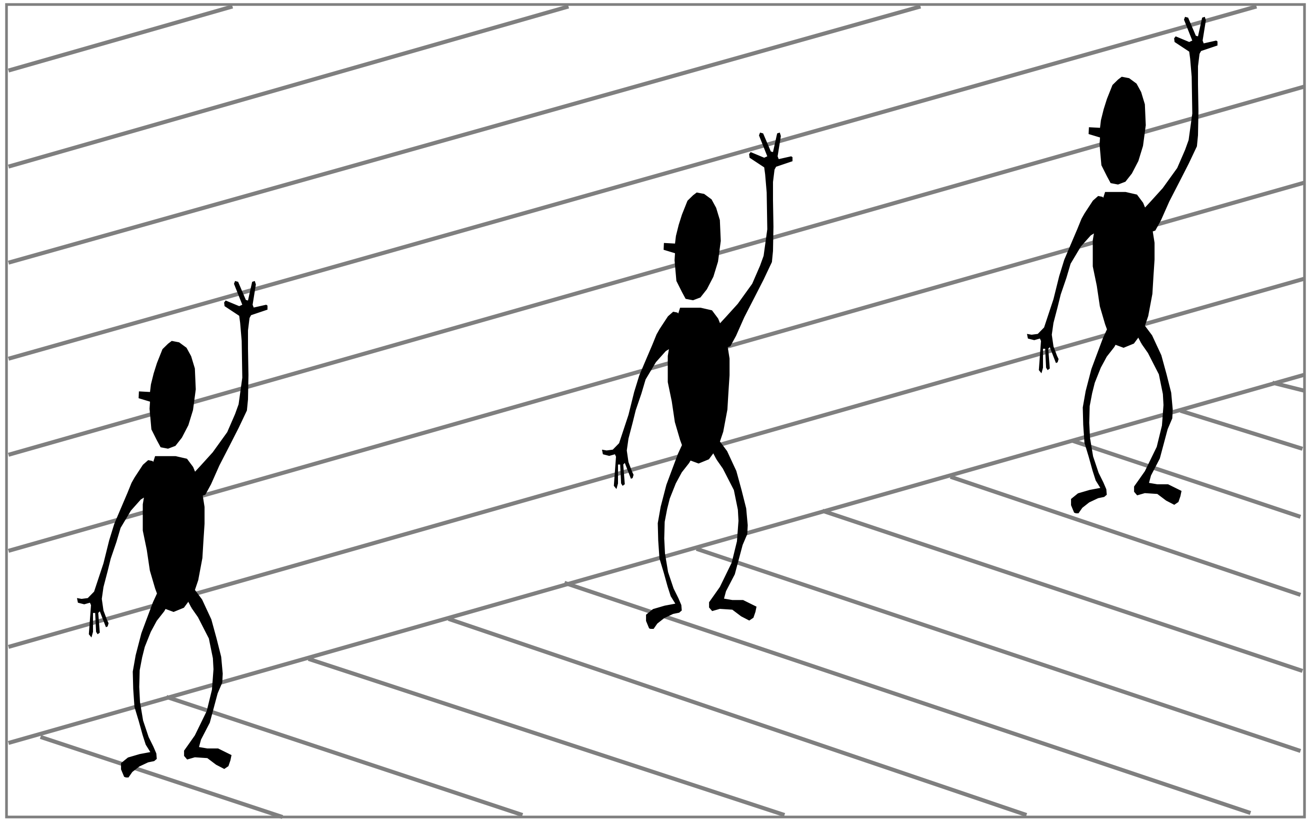


Perspective cues



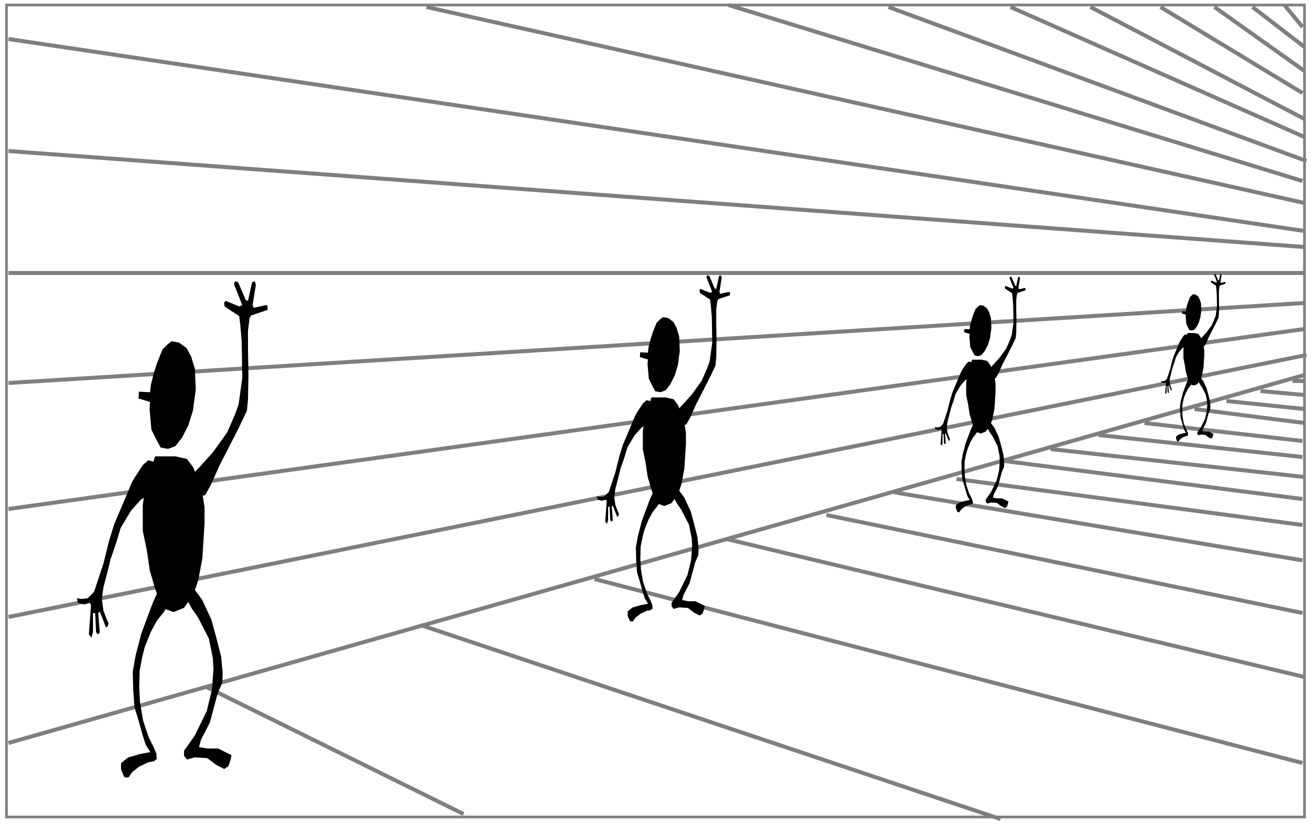
Slide by Steve Seitz

Perspective cues



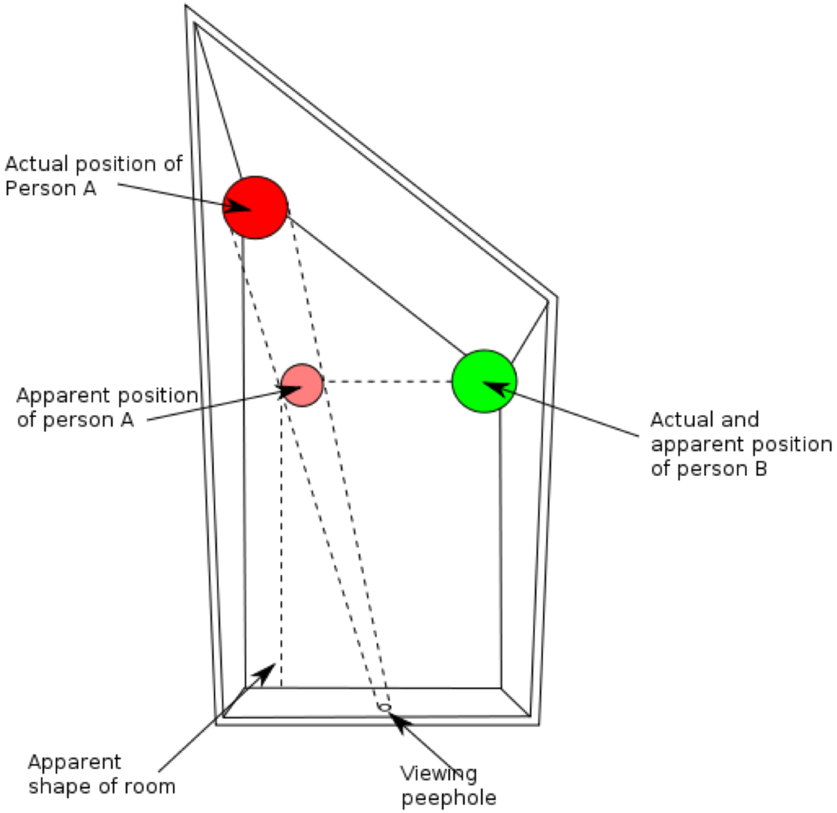
Slide by Steve Seitz

Perspective cues



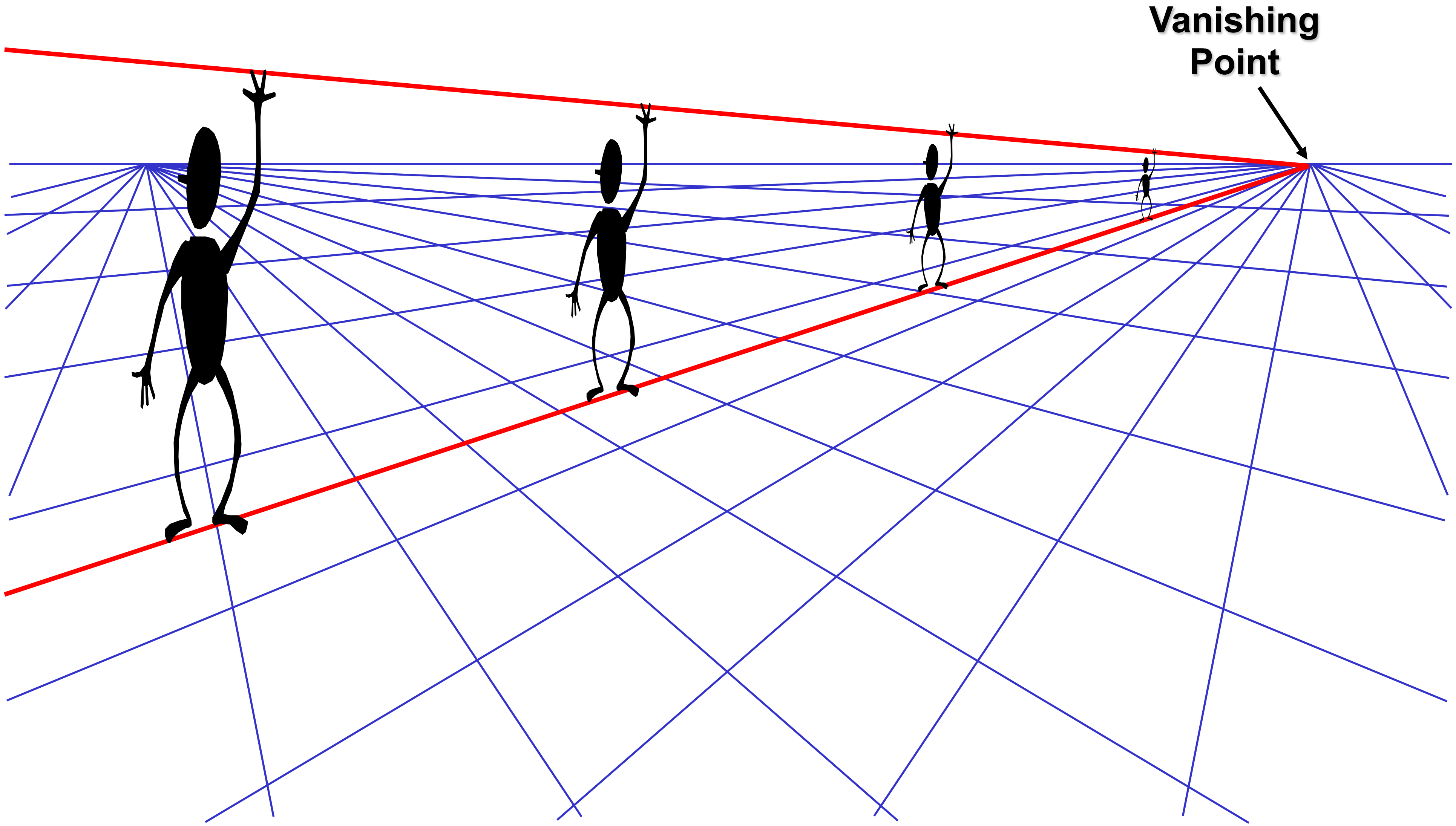
Slide by Steve Seitz

Ames Room

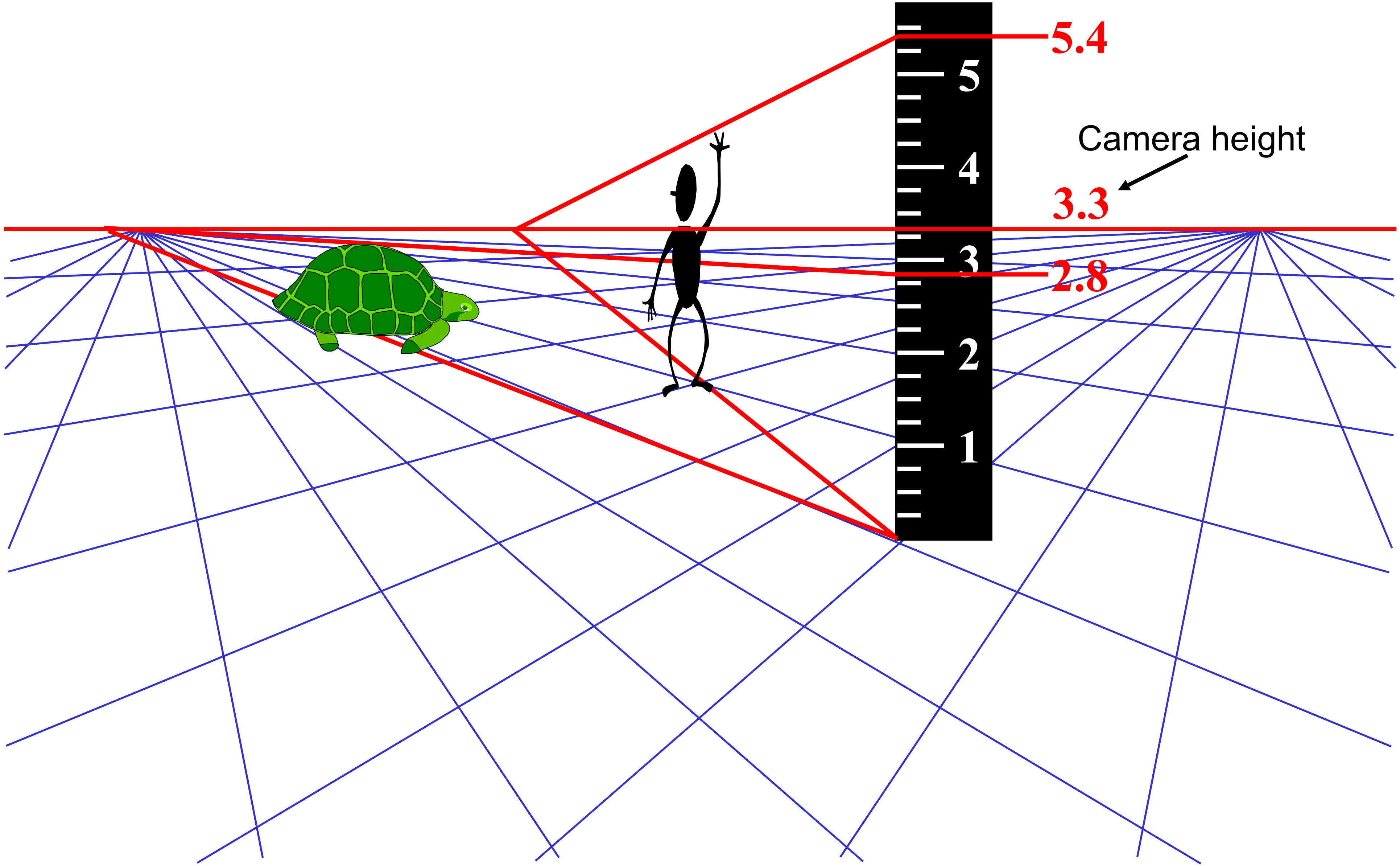

Actual position of Person A
Apparent position of person A
Actual and apparent position of person B
Apparent shape of room
Viewing peephole

Slide by Steve Seitz

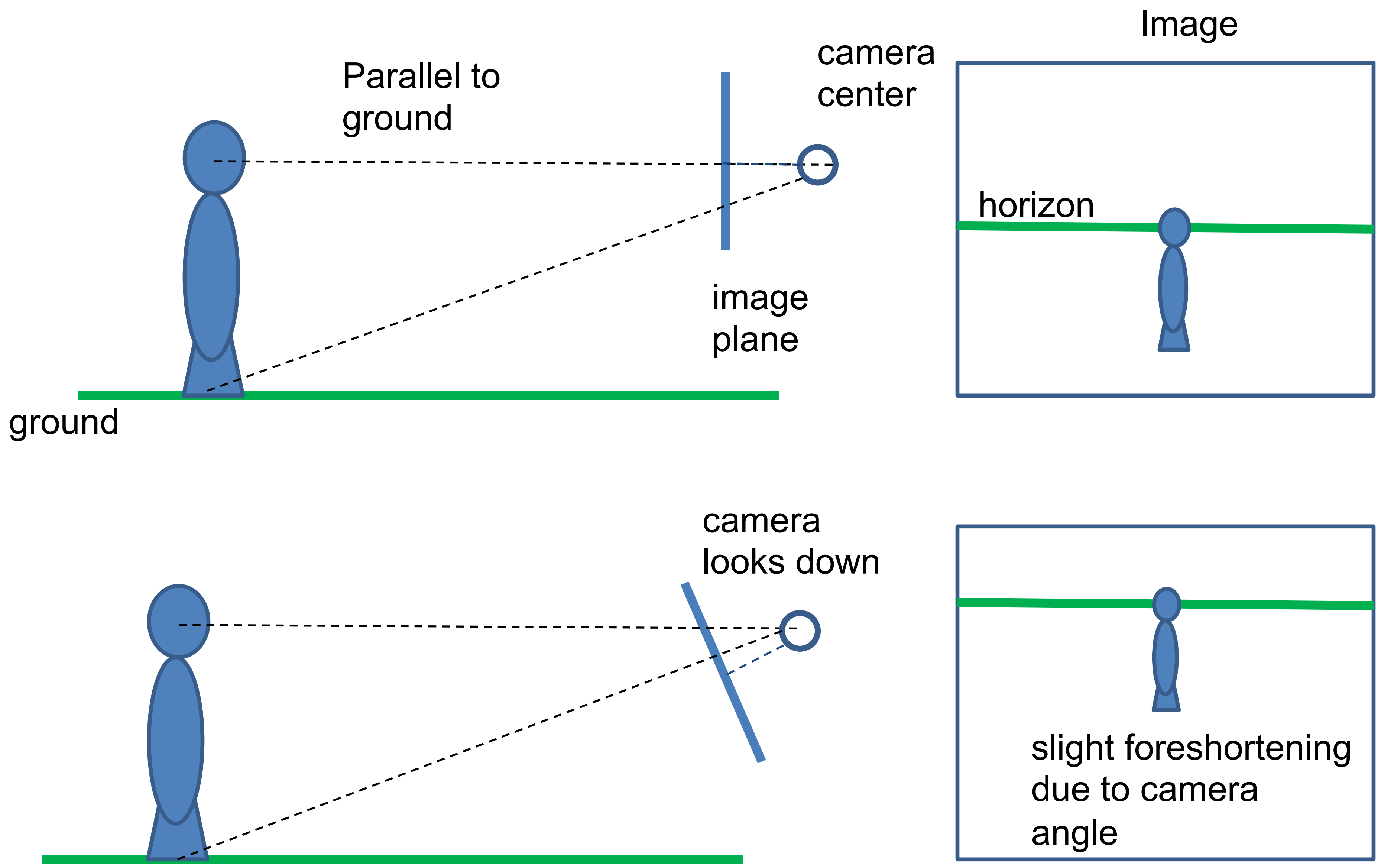
Comparing heights



Measuring height

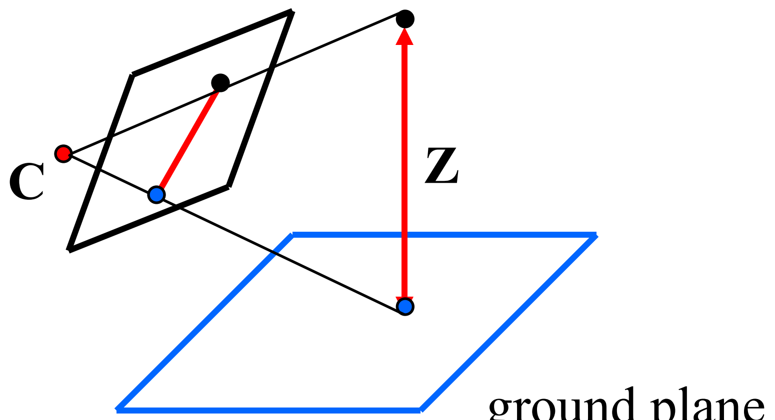


Two views of a scene



Which is higher – the camera or the parachute?

Measuring height without a giant ruler



Compute Z from image measurements

- Need a reference object

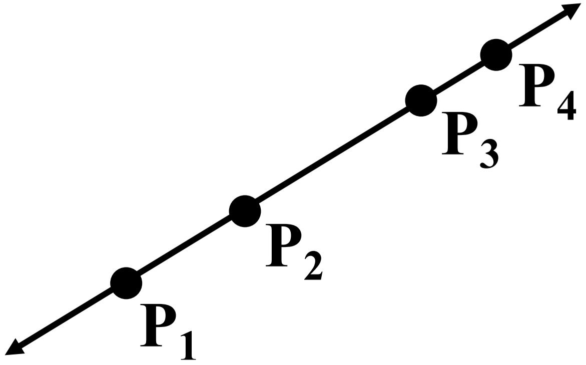
Slide by Steve Seitz

The cross ratio

A Projective Invariant

- Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points



$$\frac{\left\|\mathbf{P}_3 - \mathbf{P}_1\right\| \left\|\mathbf{P}_4 - \mathbf{P}_2\right\|}{\left\|\mathbf{P}_3 - \mathbf{P}_2\right\| \left\|\mathbf{P}_4 - \mathbf{P}_1\right\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

$$\frac{\left\|\mathbf{P}_1 - \mathbf{P}_3\right\| \left\|\mathbf{P}_4 - \mathbf{P}_2\right\|}{\left\|\mathbf{P}_1 - \mathbf{P}_2\right\| \left\|\mathbf{P}_4 - \mathbf{P}_3\right\|}$$

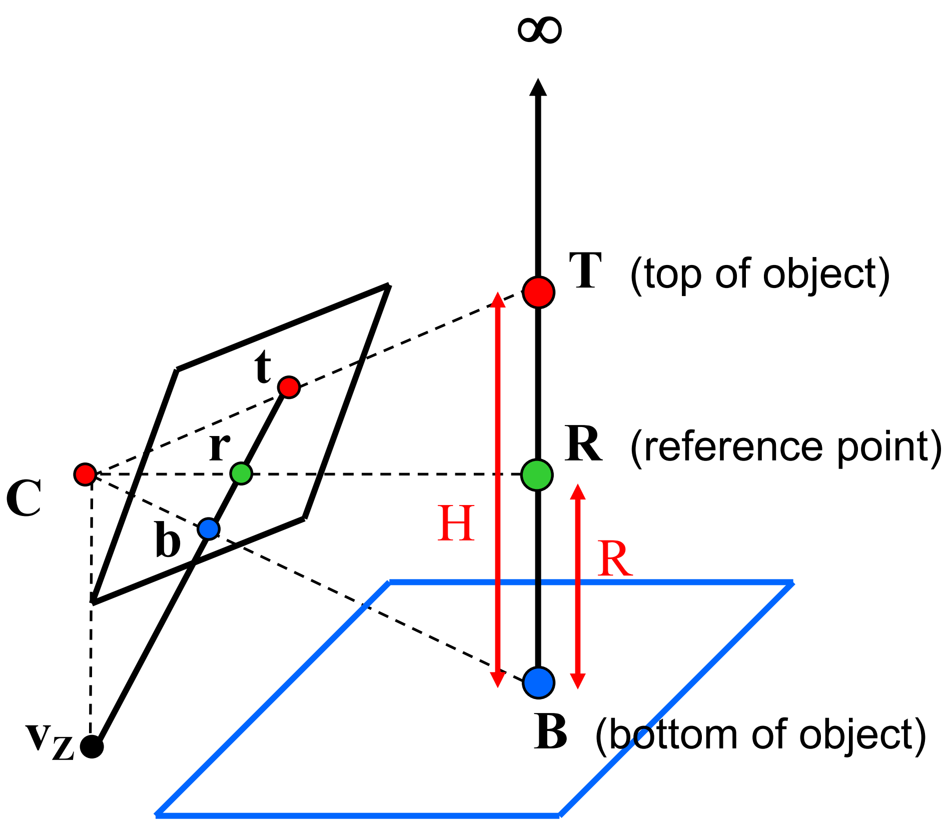
Can permute the point ordering

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Slide by Steve Seitz

Measuring height



$$\frac{\|\mathbf{B}-\mathbf{T}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{B}-\mathbf{R}\|\,\|\infty-\mathbf{T}\|}=\frac{H}{R}$$

scene cross ratio

$$\frac{\|\mathbf{b}-\mathbf{t}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{b}-\mathbf{r}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

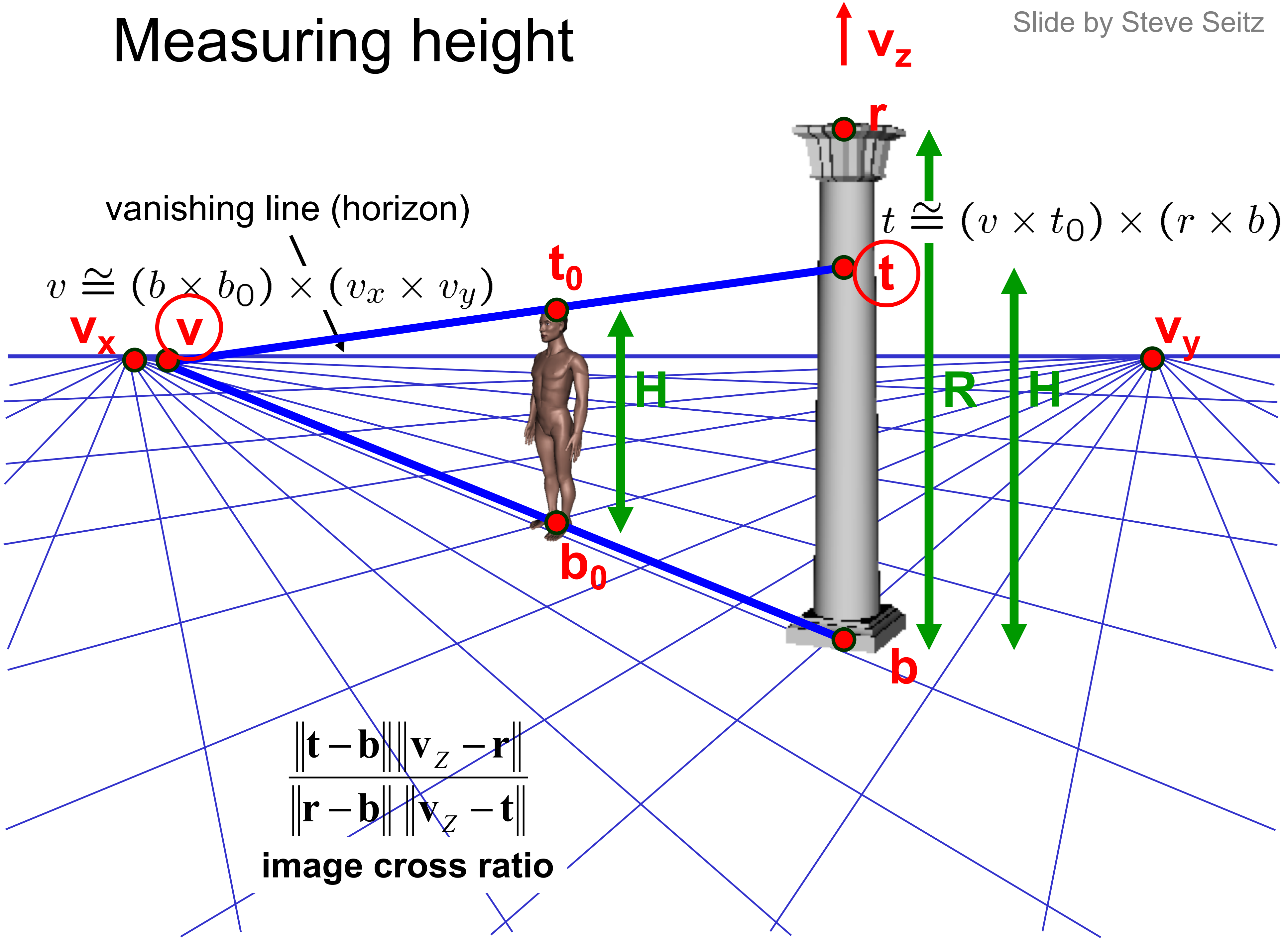
image cross ratio

scene points represented as $\mathbf{P}=\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$ image points as $\mathbf{p}=\begin{bmatrix}x\\y\\1\end{bmatrix}$

Slide by Steve Seitz

Measuring height

Slide by Steve Seitz

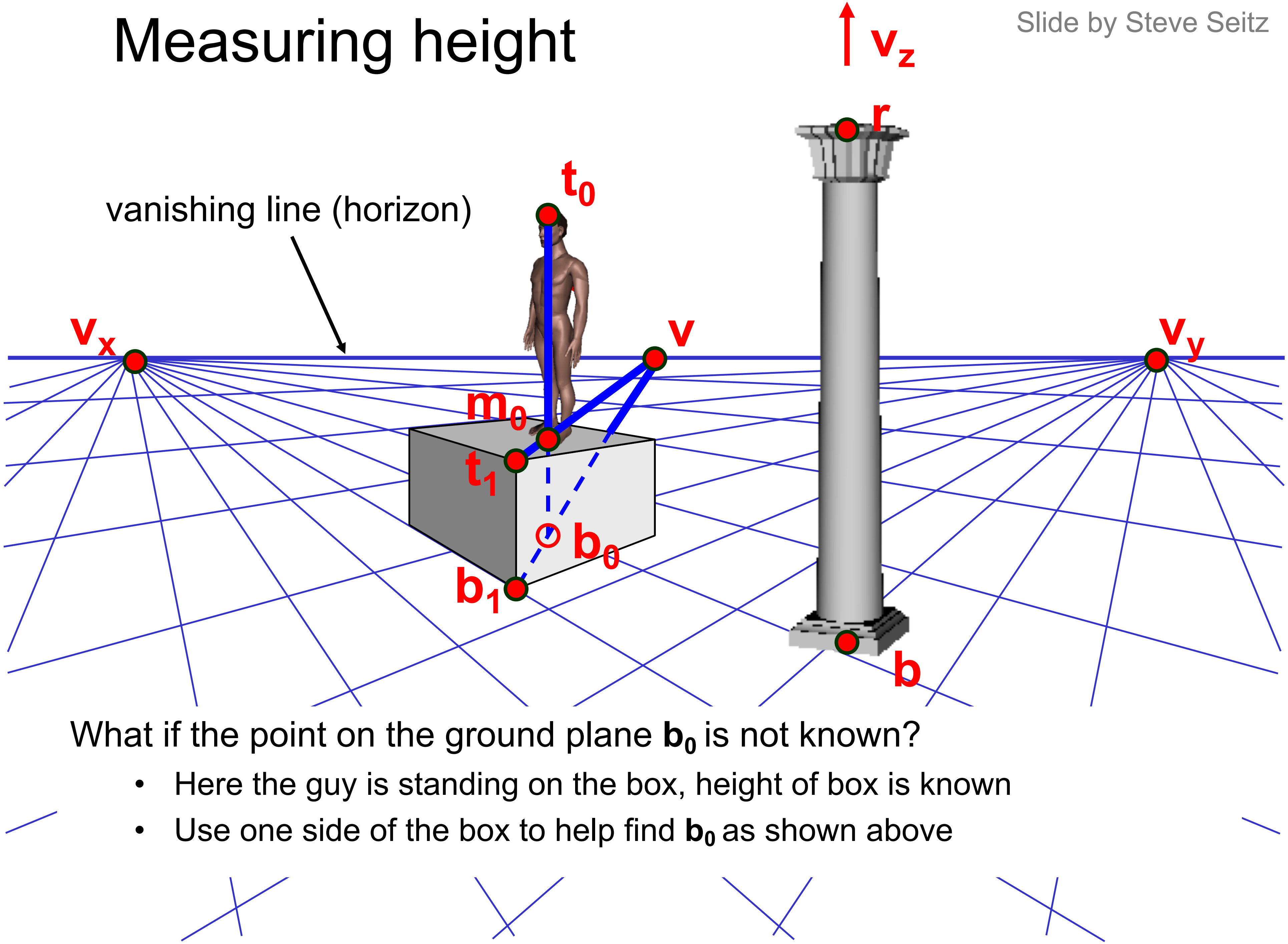


$$\frac{\|\mathbf{t}-\mathbf{b}\| \|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\| \|\mathbf{v}_Z-\mathbf{t}\|}$$

image cross ratio

Measuring height

Slide by Steve Seitz



What if the point on the ground plane $\mathbf{b_0}$ is not known?

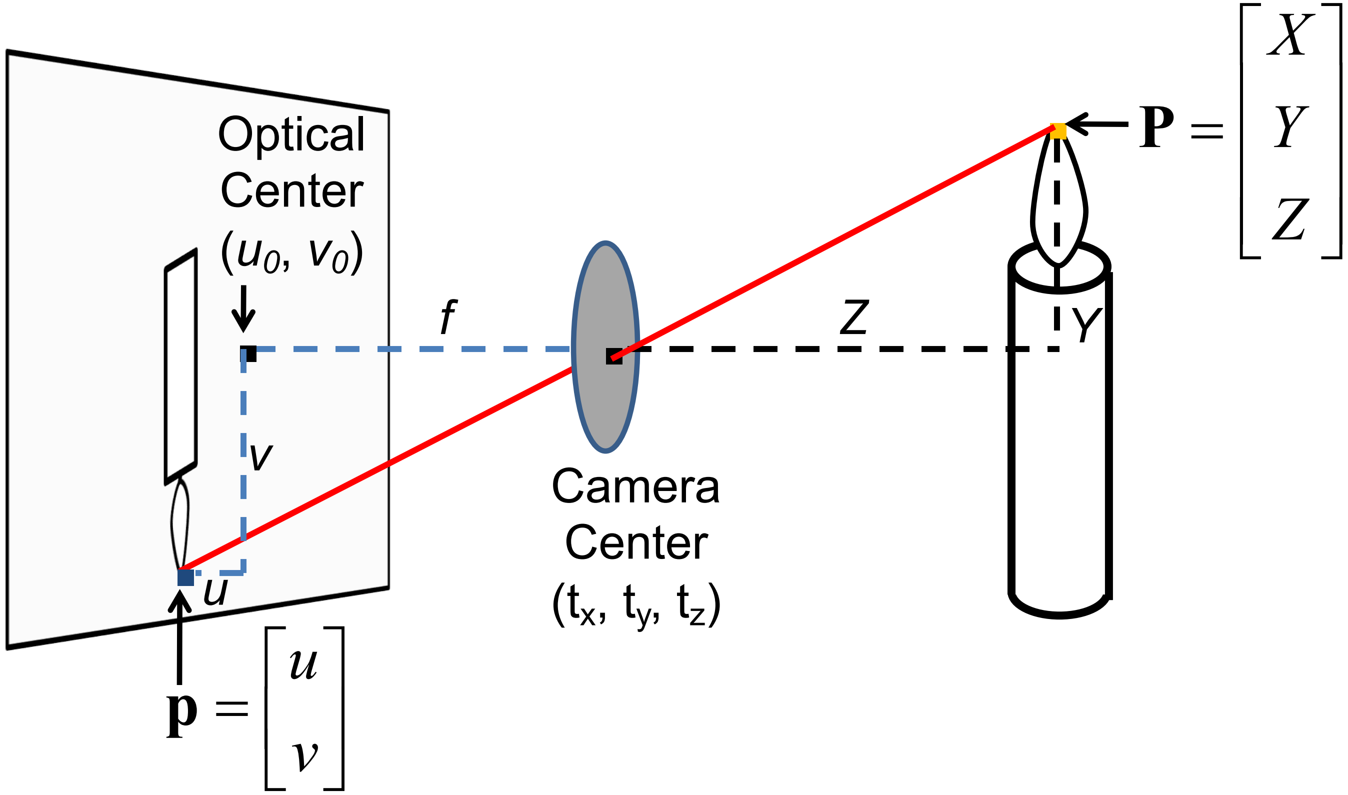
- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find $\mathbf{b_0}$ as shown above

Take-home question

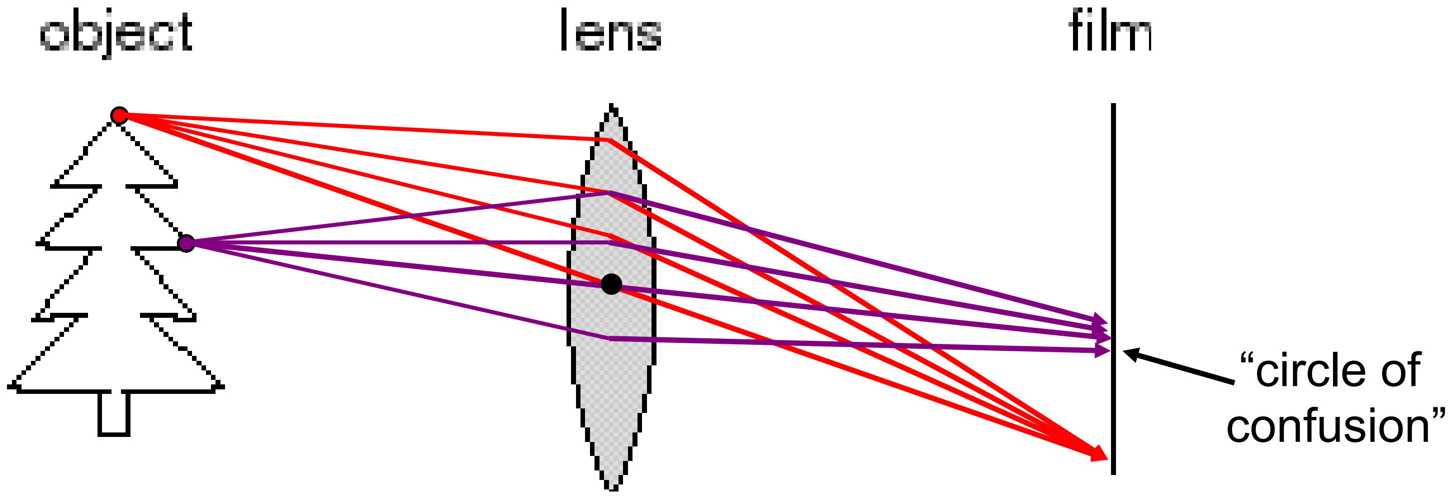
Assume that the man is 6 ft tall

– What is the height of the front of the building?
– What is the height of the camera?

Beyond the pinhole: What about focus, aperture, DOF, FOV, etc?

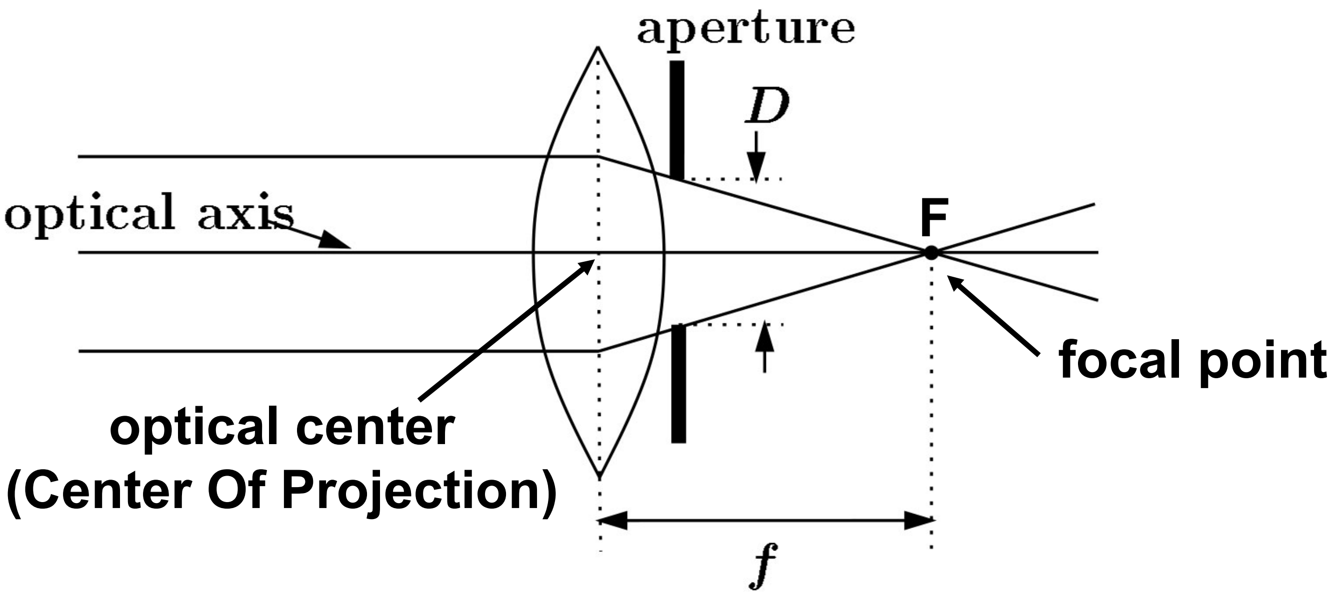


Adding a lens



- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    - other points project to a “circle of confusion” in the image
  - Changing the shape of the lens changes this distance

Focal length, aperture, depth of field

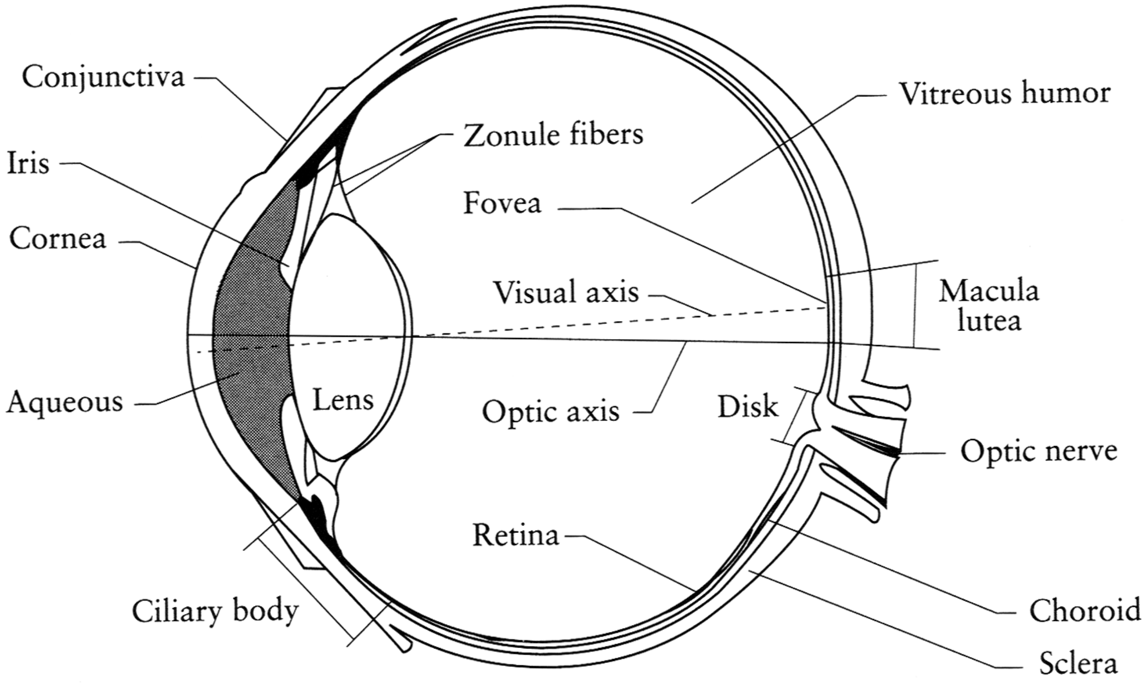


A lens focuses parallel rays onto a single focal point

- focal point at a distance *f* beyond the plane of the lens
- Aperture of diameter D restricts the range of rays

Slide source: Seitz

The eye



The human eye is a camera

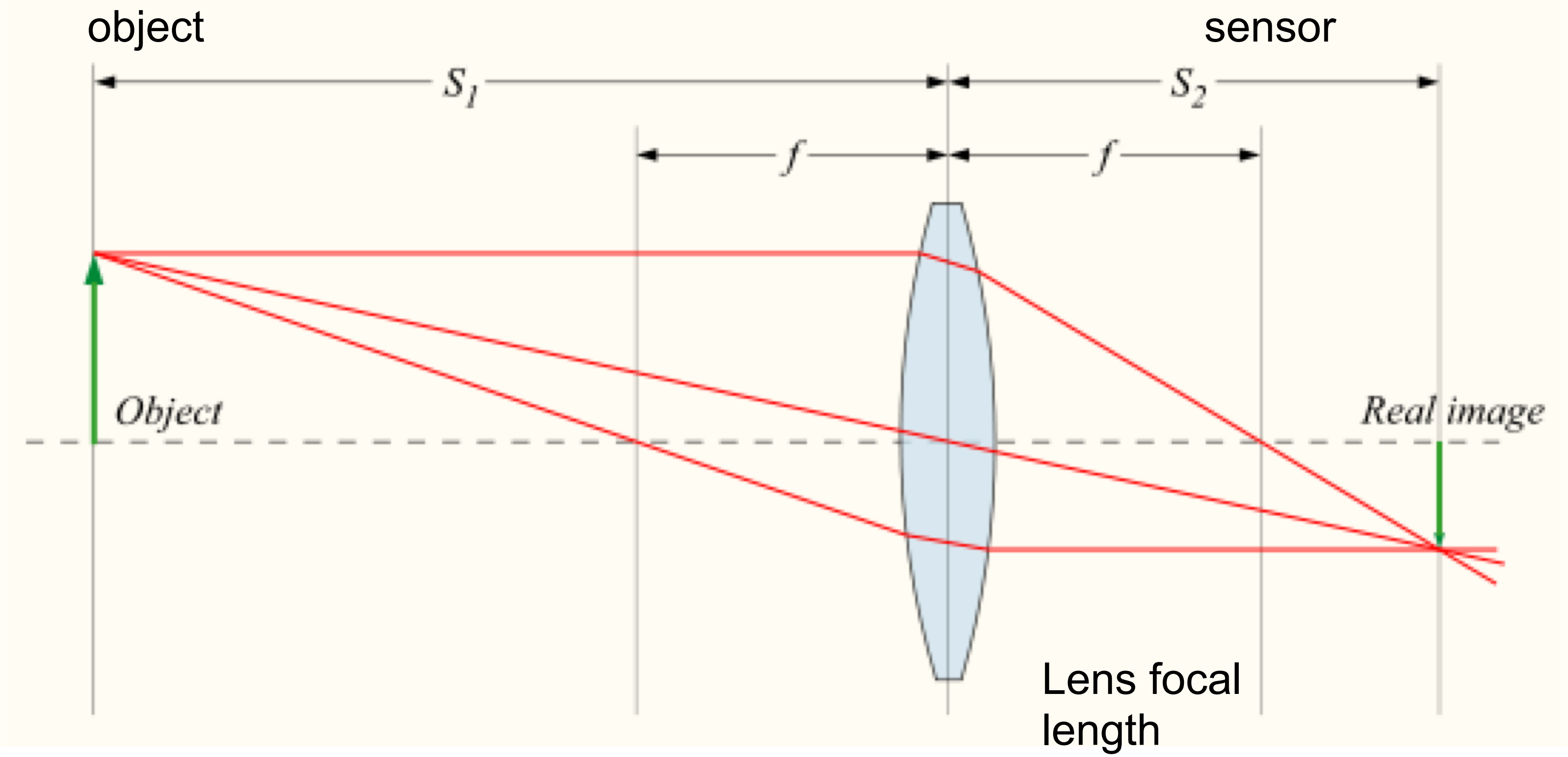
- **Iris** - colored annulus with radial muscles
- **Pupil** - the hole (aperture) whose size is controlled by the iris

Figure: Fig. 1.1, Principles of Digital Image Synthesis, Volume 1. Andrew Glassner

Focus with lenses

Distance to object

Distance to sensor



Lens focal length

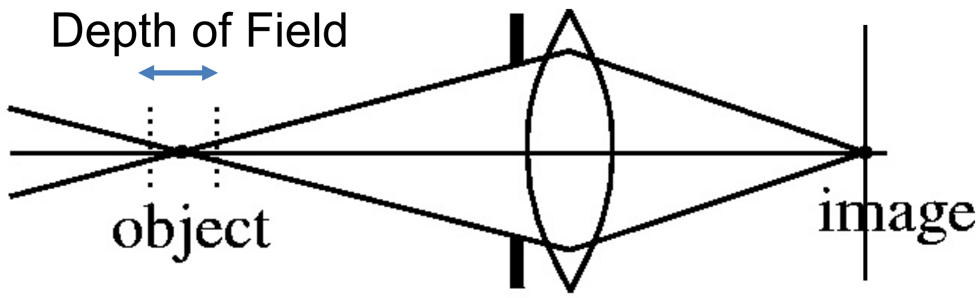
Equation for objects in focus

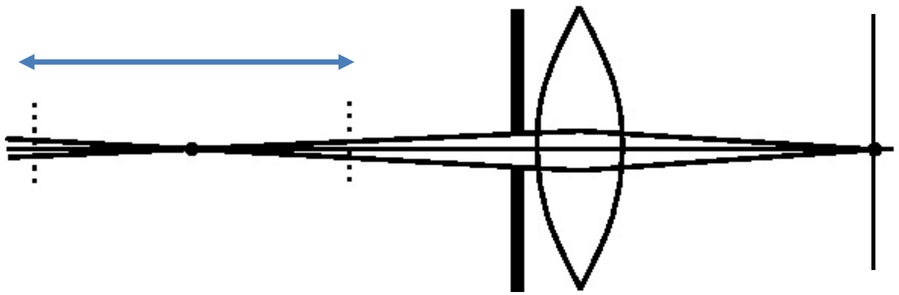
$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$

Source: http://en.wikipedia.org/wiki/File:Lens3.svg

Slide source: Seitz

The aperture and depth of field





f / 5.6

f / 32

- Changing the aperture size or focusing distance affects depth of field
  - f-number (f/#) =focal_length / aperture_diameter (e.g., f/16 means that the focal length is 16 times the diameter)
  - When you change the f-number, you are changing the aperture
- Depth of Field = range around focused distance that leads to smaller than threshold circle of confusion

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

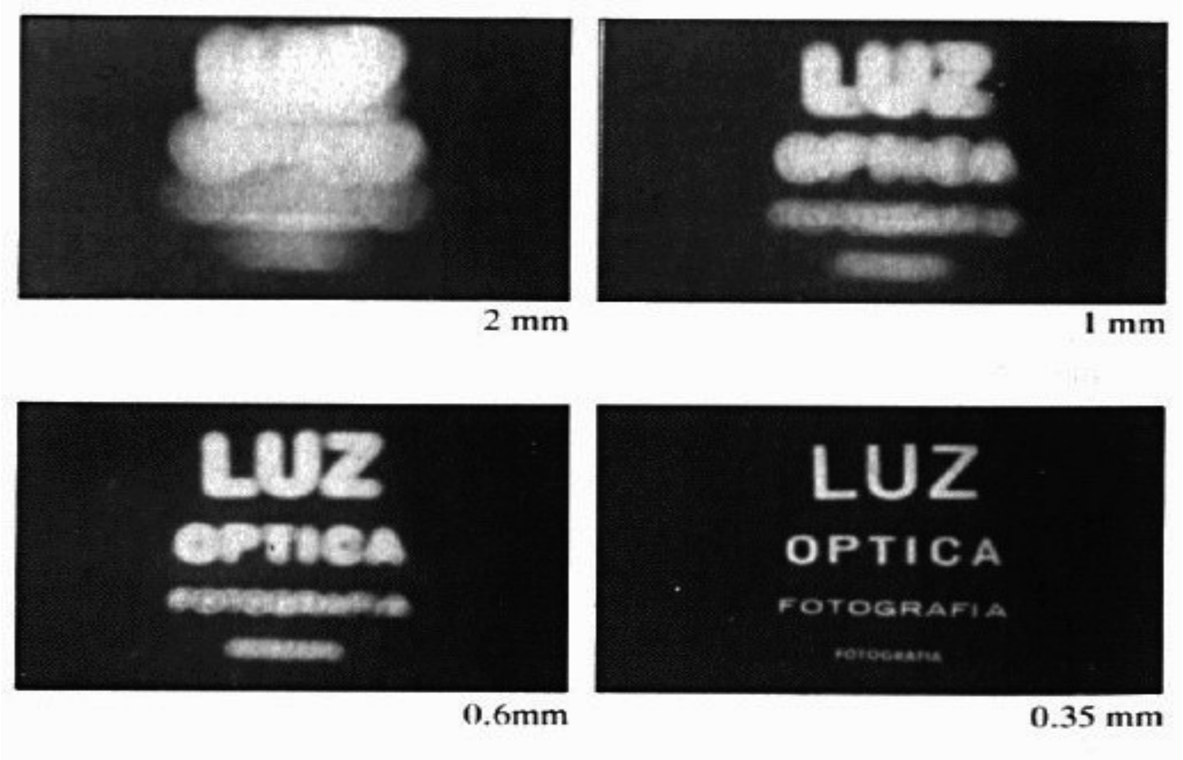
Photo credit: Philip Greenspun

Varying the aperture

Large aperture = small DOF

Small aperture = large DOF

Shrinking the aperture



Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects

Figure: *Optics*. Eugene Hecht

Shrinking the aperture

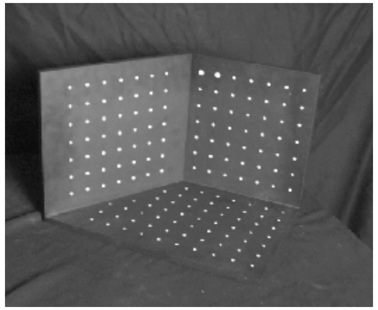
Figure: *Optics*. Eugene Hecht

The Photographer's Great Compromise

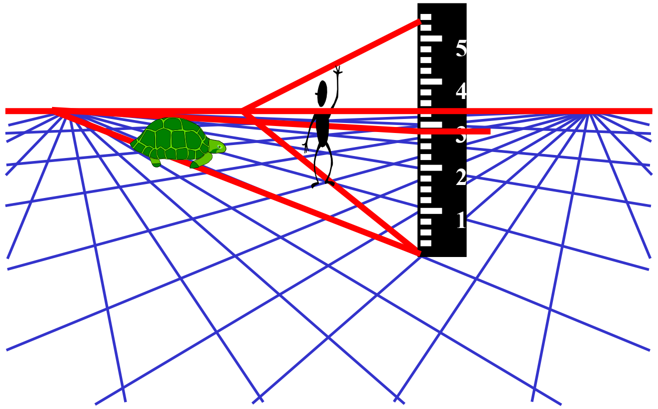
What we want	How we get it	Cost
More spatial resolution	Increase focal length	Light, FOV
	Decrease focal length	DOF
Broader field of view		
	Decrease aperture	Light
More depth of field	Increase aperture	DOF
	Shorten exposure	Light
More temporal resolution		
	Lengthen exposure	Temporal Res
More light		

Things to remember

- Can calibrate using grid or VP



- Can measure relative sizes using VP



- Effects of focal length, aperture + tricks

Next class

- Go over take-home questions from today
- Single-view 3D Reconstruction